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Product design partitions with complementary components

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Abstract

This paper combines work on modularity-in-design with results from economics on supermodular functions. I develop a model of product design in which a set of complementary components is partitioned for the purposes of product development. The model focuses on how the extent of the complementarity between components affects the optimal partitioning of components. The primary findings of the analysis are as follows: (1) Reductions in the cost of across-design-group communication make finer product design partitions more attractive, while reductions in the cost of within-group communication favor coarser partitions. (2) Firms should group components for which the next-generation design is relatively more uncertain together. (3) Firms can learn about the extent of the complementarity between components by choosing an arbitrary partition of the components and using the mix-and-match feature of 'modular' design [Baldwin, C.Y., Clark, K.B., 1994. Modularity-in-design: an analysis based on the theory of real options. Working Paper 93-026, Harvard Business School] to infer the interactions between the performance dimensions. (4) The problem of selecting an optimal partition of components is NP-complete. © 1999 Elsevier Science B.V. All rights reserved.

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1. Introduction

Product development has recently been identified as an important source of competitive advantage for business firms. Both journalists and management scientists have studied development teams for clues as to what makes for successful product introductions, with a resulting proliferation of anecdotes and case studies illustrating that how a product is partitioned into subsystems can have important effects on development performance. This

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paper presents an economic model that identifies factors that influence how firms should divide the labor of product design.

The analysis proceeds by joining together two recent strands of formal literature, one from management and one from economics. In the management literature, Baldwin and Clark (1994) analyze the effects of 'modular design' on product development performance. A product is designed in a 'modular' fashion if the work of development is partitioned among designers (or design teams) who work independently. The modules are connected by standardized interfaces that permit designers to 'mix and match' modules to find the design that maximizes some measure of performance. Baldwin and Clark apply the theory of real options to show that the 'mix and match' feature of modular design can dramatically speed the rate of performance improvement. They argue that the costs of testing the myriad products made possible by modular design may limit the effectiveness of modular design.

Another important cost that may limit the effectiveness of modular design is the cost of coordinating the actions of designers of different modules. The process of designing good interfaces and the restrictions placed on the set of feasible designs by the interfaces developed are likely to be more costly when the modules are more highly interrelated. Hence the interrelatedness of the modules affects the costs of coordination across modules. In this paper, I model the importance of coordination in product design using results on supermodular functions developed in economics by Milgrom and Roberts (1990, 1995).

I develop a model of product design in which a set of complementary components is partitioned for the purposes of product development. The model focuses on how the extent of the complementarity between components affects the optimal partitioning of components. The primary findings of the analysis are as follows: (1) Reductions in the cost of across-design-group communication make finer product design partitions more attractive, while reductions in the cost of within-group communication favor coarser partitions. (2) Firms should group components for which the next-generation design is relatively more uncertain together. (3) Firms can learn about the extent of the complementarity between components by choosing an arbitrary partition of the components and using the mix-and-match feature of modularity to infer the interactions between the performance dimensions. (4) The problem of selecting an optimal partition of components is NP-complete.

The paper proceeds as follows: Section 2 discusses related literature on organization design, product development and supermodular functions. Section 3 outlines a model of modular product development. Section 4 presents results on factors affecting the fineness of firms' product development partitions. Section 5 analyzes the question of how firms might learn about the interrelatedness of its components. Section 6 analyzes the computational feasibility of optimal design partitions. Section 7 summarizes and concludes.

2. Related literature

2.1. Studies of organization design

The idea that organizations should be structured to group related activities together has been an important theme in the study of organization for many years. March and Simon

(1958), for example, provide a detailed discussion of what they refer to as the 'theory of departmentalization.'[1] March and Simon highlight the relationship between the interdependencies among organizational subunits and the uncertainty affecting the optimal plan of action. They argue that grouping two interdependent subunits together is necessary only if unforeseen contingencies may necessitate coordinated action on the part of the two subunits. If two subunits are interdependent, but there is no uncertainty regarding the optimal action by the two units, then the correct actions (which take account of the interdependencies) can be worked out in advance. This obviates the need to communicate ex post and means the subunits need not be grouped together. March and Simon also point out that activities can be interdependent in many different dimensions, and that this fact greatly complicates attempts to find a suitable partition of activities.

Economists have attempted to formalize many of these insights in the team theory literature.[2] For example, Cremer (1980) analyzes a team-theoretic model of a firm that is composed of a set of 'shops.' Each shop can produce a variety of products and each faces random demand and random costs of production. The cost-minimizing production plan for this firm equates the marginal costs of production for each product across shops — if a shop's production of a particular product exceeds that shop's demand, then the output is transferred to another shop where demand exceeds production. Cremer rules out the efficient plan by stipulating that the firm's shops must be partitioned into a set of 'services' and that interservice transfers of production must be determined prior to the resolution of demand and cost uncertainty. Intraservice transfers are assumed to be feasible after the resolution of uncertainty. Cremer then analyzes the optimal partition of shops into services. The result is that the optimal organization minimizes a measure of the variability of the interservice transfers that would have been made if the cost-minimizing plan had been feasible. Cremer's model therefore provides a formalization of March and Simon's argument that the nature of the uncertainty as to the optimal plan is an important determinant of the optimal division of activities in firms.

2.2. *Analyses of product design*

Related ideas have been developed in the product design context by von Hippel (1990). He assumes product innovation is too complex to be carried out by a single worker, implying the work of product design must be partitioned into manageable-sized tasks. von Hippel's thesis is that how work is decomposed affects the amount of interfunctional problem solving that must be done during the design process. He argues that design projects should be partitioned to minimize the cost of coordinating activities across modules and suggests that in two ways firms may achieve this: (1) by facilitating

[1] Chandler (1962) and Simon (1973) have also contributed to a mostly informal literature on this topic.

[2] The leading work in this area is Marschak and Radner (1972). Radner (1992) applies team theory to analyze the efficiency of hierarchical structures and summarizes more recent work in the area. Team-theoretic analyses typically ignore the problem of providing incentives within organizations, focusing instead on issues relating to information flows and coordination. This distinguishes the literature from agency-based theories of the firm. My analysis falls under the heading of team theory, as I do not consider the problem of providing incentives to product design workers.

interfunctional communication, and (2) by rearranging the assignment of tasks to modules to reduce interdependence across functional groups.

Shirley (1990) provides an example of the 'mix-and-match' feature of modular design with his study of John Deere and Company's effort to redesign a family of hydraulic cylinders. The Deere engineers first selected a single core product and partitioned it into subsystems connected by standardized interfaces. Teams of specialists then designed each subsystem to fit the predetermined interface specifications. Other members of the family of cylinders were then developed by altering one or more subsystem. The interfaces linking the modules allowed the engineers to change one subsystem without requiring wholesale revisions in the rest of the system.

Shirley builds on this case study by developing a model of how firms choose core products that share design features with other potential products, so that similar design principles can be applied again and again. He frames the choice of core product as an integer programming problem and parameterizes the degree of similarity between a possible core product and other products. While Shirley takes a similar approach to mine in taking technological interactions between potential subsystems as given, he makes no attempt to model explicitly the costs of splitting apart products into modules.

Baldwin and Clark develop a model of successive generations of a product to show how modular design can speed the rate of technological change and increase product variety. Efficiency is increased because modularity allows innovations to a subsystem to be integrated into the product without a complete change in design. In addition, the 'mix-and-match' feature of modular design permits engineers to develop niche products by altering just a few components. For instance, the modular design of the Toyota Crown meant that consumers could select from up to 114 different varieties. Rather than offering costs of coordination failure as a limiting factor for modular design, Baldwin and Clark argue that the cost of testing the myriad potential products generated by modular design processes is what prevents firms from infinitely subdividing their products. They do note that splitting a design effort into appropriate modules requires extensive ex ante 'product knowledge', but their model does not capture formally the firm's problem of decomposing products into subsystems.

2.3. Supermodularity and coordination

One way in which Baldwin and Clark's framework could be expanded is through explicit consideration of the factors affecting how firms determine which components are grouped together. Baldwin and Clark take this partition of components as given in their analysis, noting that this process of modularization is difficult and requires what they refer to as 'deep product knowledge.' The firm must understand the interactions between components and modularize with an eye toward minimizing the costs of coordinating the actions of designers of different modules.

New approaches to modeling the importance of coordination have recently been developed in economics by Milgrom and Roberts. Their analyses rely heavily on the mathematics of supermodular functions. A function f: $R^n \rightarrow R$ is supermodular if, for all a and b in R^n, $f(a \vee b) - f(b) \geq f(a) - f(a \wedge b)$, where $a \wedge b$ (referred to as the *meet* of a

and b) denotes the component-wise minimum of the vectors a and b, and $a \vee b$ (the *join* of a and b) is the component-wise maximum.

Supermodularity implies that the change in $f(a)$ when any component of the vector a is increased is non-decreasing in the other components of a. If f is supermodular, the arguments of f are *complements*. An example in R^2 helps to make this point. Let $a = (x_a, y_a)$ and let $b = (x_b, y_b)$ with $x_a > x_b$ and $y_b > y_a$. If f is supermodular, then

$$f(x_a, y_b) - f(x_a, y_a) \geq f(x_b, y_b) - f(x_b, y_a).$$

This inequality states that the change in the value of f when the second component increases from y_a to y_b is larger when the first component is larger. It is easy to show that if f is supermodular and differentiable, then all the cross-partial derivatives of f are non-negative.

An approach to modeling coordination in research and development using supermodularity is suggested by a theorem presented in Milgrom and Roberts (1995).

Theorem 1 (Milgrom and Roberts). *Let f: $R^n \rightarrow R$ be a supermodular function and let $\epsilon_0, \ldots, \epsilon_n$ be a vector of i.i.d. random variables. Then, for all $x = (x_1, \ldots, x_n) \in R^n$,*

$$E[f(x_1 + \epsilon_0, \ldots, x_n + \epsilon_0)] \geq E[f(x_1 + \epsilon_1, \ldots, x_n + \epsilon_n)]. \quad (1)$$

They interpret this theorem to imply that, when complementarities are present, 'fit' is important. They write "even mistaken variations from a plan are less costly when they are coordinated than when they are made independently" (p. 186). The intuition for this theorem is as follows: Since the x_i's are complementary, the marginal return to increasing x_1 is highest when $x_2, \ldots, x_n$ are high. It is therefore costly to have x_1 low when $x_2, \ldots, x_n$ are high. Similarly, the marginal return to increasing x_1 is lowest when $x_2, \ldots, x_n$ are low. It is therefore not so valuable to have x_1 high when $x_2, \ldots, x_n$ are low. It is most valuable to have the x_i's move together[3].

In applying this theorem to the problem of product development, it is important to be precise about the sense in which supermodularity captures a notion of fit. Clark and Fujimoto (1991) stress in their study of product development processes in the automobile industry that, for many modern products, there are two distinct senses in which components of a product must fit.[4]

> Product integrity has both internal and external dimensions. Internal integrity refers to consistency between the function and structure of a product – e.g., the parts fit well, components match and work well together, layout achieves maximum space efficiency. External integrity is a measure of how well a product's function, structure, and semantics fit the customer's objectives, values, production system, lifestyle and self-identity.

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[3] Prat (1996) proves a similar result in a different context. He studies a firm that must choose the information available to each of its agents and shows that if the function mapping agents' actions into payoffs is supermodular, then an optimal information structure is for each agent to have the same partition on the state space. Prat offers intuition similar to that given by Milgrom and Roberts: when complementarities are present, it is important to establish 'the right relation among choice variables' (p. 189).

[4] Clark and Fujimoto's study, which is based on extensive field research in the worldwide automobile industry, presents numerous examples of firms facing tradeoffs of the type studied in this paper.

The model presented so far has not specified what generates the need for performance dimensions of various components to fit. Both types of product integrity, as defined by Clark and Fujimoto, could apply. Physical fit could generate complementarities between components as in the example above, but so could the need to coordinate various parts of a vehicle to achieve a certain style. As an example, automobile gasoline and oil gauges do not have a significant physical interaction, but their attributes may need to be coordinated to present a unified design theme to the driver.

3. The model

In this section, I extend Baldwin and Clark's model of product development, in which research and development activity results in a random 'shock' to each component, by assuming the function mapping component performance to product performance is supermodular. This implies that the firm improves coordination and achieves a higher payoff if it can subject each component to the same shock. I develop a simple model of the trade-off between specialization and coordination by assuming the mean of the research shock affecting a component is higher when there are fewer other components in the same design group. However, I assume research shocks are more highly correlated within design groups than across. To realize specialization economies, the firm divides the labor of research and development by partitioning components into subsystems. While this splitting apart of the product allows R&D workers to manage complexity, it comes at the cost of reduced coordination across design groups.

I first assume the product is composed of N different components that must be designed and assembled into a unified whole. For each component, there is an associated 'performance measure.' The state of a component according to the performance measure is represented by a real number. Second, I describe 'product performance' as a real number that is a non-decreasing function of the performance dimensions of the components. Third, I assume the function mapping components into product performance is supermodular. This assumption implies that the individual components' performance dimensions are complements in the product performance function and that how the components 'fit' together is important. I then model the research and development process as generating random shocks to the performance dimensions of the various components. Finally, I state the firm's optimization problem and show that how a firm partitions components into design groups can affect product development performance.

A firm undertakes design of a product consisting of N components. For each component $i \in \{1, \ldots, N\}$, there is a 'performance measure' that affects the overall performance of the product. I assume component i's 'performance' can be measured by a real number, which I denote as x_i.

I assume that performances of components are complements in the overall performance of the product. The complementarity assumption states that the marginal improvement to overall performance from increasing the performance of one component is increasing in the performance of the other components. Mathematically, the assumption requires that the function mapping components' performances into overall performance is supermodular. I assume this function is differentiable and the derivative of product performance with respect to component performance measures x_i and x_j is a constant, denoted by $\delta_{ij} \geq 0$.[5] I refer to δ_{ij} as the *complementarity parameter* for components i and j. The overall performance of the product is defined as

$$X = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \delta_{ij} x_i x_j.$$

I assume the firm is endowed with an initial set of component designs at time t from which it produces an initial version of the product. At time $t+1$, the firm attempts to improve the product by engaging in research and development.[6] To capture the idea that experimentation by nature yields uncertain results, I assume that a component's performance at time $t+1$ is the sum of its performance at time t and a random research shock.

To develop this idea more formally, I introduce some additional notation. Let the overall performance of the product at time t be X_t. For component i, performance at time t is denoted by x_{it}. The research process for component i can then be summarized by the following:

$$x_{it+1} = x_{it} + \epsilon_{it+1},$$

where ϵ_{it+1} is a random variable with mean μ_i and variance σ_i^2. A second generation of the product can then be created by fitting the new components together to get a product with overall performance

$$X_{t+1} = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \delta_{ij} x_{it+1} x_{jt+1}.$$

I assume that expected research productivity on component i is higher when it is in a design group with fewer other components. This assumption is driven by the hypothesis that design teams can achieve larger gains from specialization when they have fewer components to design. To capture the costs of imperfect coordination, I assume that research shocks to components in different design groups are less positively correlated, while shocks to components in common groups are more highly correlated. This assumption captures the intuition of the theorem from Milgrom and Roberts presented above.

A *design group partition* is a collection of subsets $\mathcal{G}_1, \ldots, \mathcal{G}_m$ of the set of components $\{1, \ldots, N\}$ such that $\bigcup_{i=1}^{m} \mathcal{G}_i = \{1, \ldots, N\}$ and $\mathcal{G}_i \cap \mathcal{G}_j = \emptyset$ for all $i \neq j$. I define G: $\{1, \ldots, N\} \to \{\mathcal{G}_1, \ldots, \mathcal{G}_m\}$ as follows: $G(i) = \{\mathcal{G}_j | i \in \mathcal{G}_j\}$. The function G takes a component as an argument and returns the design group of which the component is a member. Since expected research productivity on component i is a function of the number of components in component i's design group, I have $\mu_i = \mu(\#G(i))$, where μ: $N \to R \cup \{-\infty\}$ is non-increasing. I allow the expected research productivity to be

[5] This is a significant restriction: the definition of supermodularity requires only that the cross-partials are positive, not that they are constant. This restriction allows me to consider cases in which the shocks to different components are positively, but not perfectly, correlated. Note that the Milgrom and Roberts theorem discussed earlier requires that the shocks on the left hand side of (1) be identical and the shocks on the right hand side be independent. By choosing a specific functional form, I am able to generalize this statement to cases where the shocks are more highly correlated on the left than the right.

[6] While the discussion centers around improving an existing product, there is nothing in the model to preclude analyzing a firm trying to partition the work of designing an entirely new product.

$-\infty$ to model the possibility that design groups of certain sizes may be infeasible. For components i and j, research shocks are more correlated if the components are in the same design group and less correlated otherwise. Let $0 \leq \rho_{\mathrm{a}} < \rho_{\mathrm{w}} \leq 1$ and suppose

$$\mathrm{Cov}(\epsilon_i, \epsilon_j) = \begin{cases} \rho_{\mathrm{w}}\sigma_i\sigma_j & \text{if } G(i) = G(j) \\ \rho_{\mathrm{a}}\sigma_i\sigma_j & \text{otherwise.} \end{cases}$$

The parameter ρ_{w} denotes 'within-design-group' correlation of research shocks, while ρ_{a} denotes the 'across-design-group' correlation.

I can now state the firm's problem in choosing a design group partition. The firm is endowed with a set of component designs with overall performance X_0 at time $t = 0$. By selecting a design group partition and engaging in product design research, the firm attempts to maximize the expected overall performance of the product at time $t = 1$, X_1.

The firm's problem can be written

$$\max_{\mathcal{G}_1,\ldots\mathcal{G}_m} X_0 + \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \delta_{ij}\{x_i\mu[\#G(j)] + x_j\mu[\#G(i)] + \mu[\#G(i)]\mu[\#G(j)]$$
$$+\sigma_i\sigma_j(\rho_{\mathrm{a}}I[G(i) \neq G(j)] + \rho_{\mathrm{w}}I[G(i) = G(j)])\}, \tag{2}$$

where $I[\cdot]$ is an indicator function returning one if its argument is true and zero otherwise. In this expression, I have factored out the overall performance of the product at $t = 0$, and expanded the term inside the summation. The first three terms inside the summation relate to the gains to specialization: since μ is non-increasing, the firm achieves higher research productivity if it chooses smaller design groups. The final two terms capture the assumption that it is more difficult to coordinate across design groups than within. Research shocks across components are more correlated if the components are in the same module.

I now present a simple example to illustrate some of the trade-offs the model is intended to capture. Suppose a product consists of four components. At time $t = 0$, each component's performance is normalized to zero. The firm selects a product design partition to maximize the overall performance of the product at time $t = 1$. If the firm chooses to keep the four components in the same design group, it achieves greater coordination between the components, but it misses out on some specialization economies. If the components are separated into distinct design groups, then design teams can specialize their efforts, but coordinating the attributes of the product is more difficult.

I write the expected overall performance of the product at time $t = 1$ as a function of the firm's choice of design group partition. Assuming a single design group and $\sigma_i^2 = \sigma^2$ for $i \in \{1, \ldots, 4\}$,

$$E[X_1 \mid 1 \text{ design group}] = E\left[\sum_{i=1}^{3}\sum_{j=i+1}^{4} \delta_{ij}x_ix_j\right] = \sum_{i=1}^{3}\sum_{j=i+1}^{4} \delta_{ij}E[\epsilon_i\epsilon_j]$$
$$= \sum_{i=1}^{3}\sum_{j=i+1}^{4} \delta_{ij}(\mathrm{Cov}[\epsilon_i, \epsilon_j] + E[\epsilon_i]E[\epsilon_j]) = (\mu(4)^2 + \rho_{\mathrm{w}}\sigma^2)\sum_{i=1}^{3}\sum_{j=i+1}^{4} \delta_{ij}. \tag{3}$$

Now suppose the firm selects two design groups: $\mathcal{G}_1 = \{1, 2\}$ and $\mathcal{G}_2 = \{3, 4\}$.

$$E[X_1 \mid 2\,\text{design groups}] = \sigma^2(\rho_\mathrm{w} - \rho_\mathrm{a})(\delta_{12} + \delta_{34}) + (\mu(2)^2 + \rho_\mathrm{a}\sigma^2)\sum_{i=1}^{3}\sum_{j=i+1}^{4}\delta_{ij}. \quad (4)$$

When there are two design groups, R&D workers are able to specialize. This results in higher expected research productivity on each component. In the equations above, the difference in specialization economies is captured by the difference between $\mu(2)^2$ and $\mu(4)^2$. The cost of this specialization is imperfect coordination between the groups. The coordination loss associated with the partition is represented mathematically by

$$\sigma^2(\rho_\mathrm{w} - \rho_\mathrm{a})(\delta_{13} + \delta_{14} + \delta_{23} + \delta_{24}),$$

which appears in Eq. (3) but not in Eq. (4). If

$$(\mu(2)^2 - \mu(4)^2)\sum_{i=1}^{3}\sum_{j=i+1}^{4}\delta_{ij} \geq \sigma^2(\rho_\mathrm{w} - \rho_\mathrm{a})(\delta_{13} + \delta_{14} + \delta_{23} + \delta_{24}), \quad (5)$$

then two design groups are preferred; otherwise the firm is better off with a single design group.

Eq. (4) gives an indication as to how a firm might select an optimal design partition. Since the coordination loss is proportional to the sum of the δ_{ij}'s for which i and j are not in the same groups, the firm wishes to group together any two components i and j for which δ_{ij} is large. The larger the cross-partial derivative between x_i and x_j in the function mapping component performance to overall performance, the higher the return to grouping i and j together.

4. Determinants of product design partitions

This section takes the technology of the firm's product as given (in the form of the complementarity parameters δ_{ij}) and presents two simple results relating to other factors that influence a firm's choice of the number of design groups. As von Hippel's analysis suggests, changes in the firm's ability to communicate information within or across design groups cause the firm to alter its design group choice. I also show how differences in the variance of research shocks across components can affect design group partitions.

Suppose the firm can purchase communication technology that allows it to improve the coordination of research activity across components. I model this technology as improving either within-design-group coordination or across-design-group coordination. Suppose that at cost $C_\mathrm{w}(\rho_\mathrm{w}; \gamma_\mathrm{w})$, the firm can purchase and install technology that allows the correlation between research shocks to components in the same design group to be ρ_w. Let γ_w parameterize the marginal cost of such communication technology:

$$\frac{\partial^2 C_\mathrm{w}}{\partial\rho_\mathrm{w}\partial\gamma_\mathrm{w}} > 0. \quad (6)$$

Similarly, suppose that the firm may purchase technology that improves across-design-group coordination at cost $C_a(\rho_a; \gamma_a)$, and that γ_a parameterizes the marginal cost of such technology:

$$\frac{\partial^2 C_a}{\partial \rho_a \partial \gamma_a} > 0. \tag{7}$$

The proposition below states how changes in γ_w and γ_a affect the firm's choice of design groups. I sidestep integer problems by assuming that the product is comprised of $N = n!$ components (for some integer n) and that the optimal design group partition always involves m equal-sized groups.

Proposition 1. *The optimal number of design groups, m, is a non-increasing function of the across-group communications parameter, γ_a, and a non-decreasing function of the within-group communications parameter, γ_w.* See Appendix for proof.

The proposition states that changes in the costs of communication technology can cause the firm to alter its product design groups. As Eq. (5) demonstrates, when a firm chooses a finer product design partition, there are two effects. Specialization economies increase, leading to higher expected research productivity. However, the smaller design groups mean the firm is less able to coordinate research activity. As the cost of across-group communication falls, the firm is better able to coordinate the actions of different groups. This makes smaller product design groups relatively more attractive. As the costs of within-group coordination fall, small design groups are at a bigger disadvantage relative to large design groups.

In practical situations, one would likely expect improvements in communication technology to reduce the costs of both within- and across-group communication. The analysis here suggests that how communication technology affects choice of design groups depends in large part on how the technology is put to use. In particular, it is important to consider how the technology affects the firm's ability to coordinate within versus across design groups. If the technology increases (reduces) this difference, then the firm should choose fewer (more) design groups.

Another factor affecting how components are split into design groups is the relative variances of the research shocks to the components. Differences in σ_i^2 across components can have implications for product design groupings.

Proposition 2. *Assume $\delta_{ij} > 0, \sigma_i^2 = \sigma_j^2 = b\sigma^2$ and $\mu(2) > -\infty$. Then, for b sufficiently large, the optimal design group partition has the property that G(i) = G(j). (See Appendix for proof).*

I illustrate the proposition using a simple example. Consider a product with four components that must be partitioned into two equal-sized design groups. Normalize $x_{i0} = 0$ for all $1 \leq i \leq 4$ and suppose $\delta_{12} = \delta_{34} = 2$, while $\delta_{13} = \delta_{14} = \delta_{23} = \delta_{24} = 1$. I assume the research associated with components 2 and 3 is more uncertain than that associated with 1 and 4. In particular, suppose the variance of ϵ_{i1} is σ^2 for $i = 1, 4$ and $b\sigma^2$ for $i = 2$, 3, with $b > 1$. Also suppose that $\rho_w = 1$ and $\rho_a = 0$.

Considering only the complementarity parameters, it is clear where the interactions among the components are strongest. Components 1 and 2 are closely related, as are 3 and 4. If the firm chooses design group partition $\mathscr{G}_1 = \{1, 2\}$ and $\mathscr{G}_2 = \{3, 4\}$. Then the expected performance of the product at time 1 is

$$E(X_1) = 4\mu(2) + \mu(2)^2 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \delta_{ij} + \sqrt{b}\sigma^2 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \delta_{ij} I(G(i) = G(j))$$
$$= 4\mu(2) + 12\mu(2)^2 + 4\sqrt{b}\sigma^2.$$

Note, however, that if instead the firm chooses $\mathscr{H}_1 = \{1, 4\}$ and $\mathscr{H}_2 = \{2, 3\}$, then the expected performance of the product at time 2 is

$$E(X_1) = 4\mu(2) + 12\mu(2)^2 + (b+1)\sigma^2.$$

Simple calculations reveal that if b is sufficiently large, then partition $\mathscr{H}$ is preferred to $\mathscr{G}$.[7] This example suggests that firms' product design groups are affected by factors other than just the technological complementarity captured by the δ_{ij}'s – uncertainty surrounding component improvements affects the potential costs of coordination as well.

The intuition for this proposition is as follows: High variance components are those for which the firm has little knowledge about the form of the next generation. For such components, it is difficult for the firm to determine ex ante how the components should fit together in the final version of the product. By grouping these components into the same design group, the firm can ensure that changes in one component are properly reflected in the other. To understand the result mathematically, first note that because δ_{ij} is positive, the marginal return to increasing x_i is highest (lowest) when x_j is high (low) as well. As the variance of ϵ_{j1} increases, it becomes relatively more likely that x_{j1} is either very high or very low. It therefore also becomes relatively more likely that the marginal return to x_i is either very high or very low. Placing x_i and x_j in the same design group when σ_j^2 increases guarantees that x_i is high when the marginal return to x_i is high.

This result – that the 'uncertainty' associated with component research affects the optimal product design partition – is in some ways similar to Cremer's conclusion that the efficient partition of shops into services minimizes a measure of variability of transfers across services. The fact that the two models, which are mathematically very different, yield similar results confirms the importance of the link between task partitioning and the uncertainty related to the tasks. There is an important sense, however, in which the analysis here is more general than Cremer's. In his model, the only relevant notion of 'relatedness' of shops is the variability of the efficient transfers between the shops. In my model, there are two distinct notions of relatedness: one arising from the complementarity parameters, and one arising from the uncertainty as to the next generation of the components. I find that the two work together to determine the optimal division of activities. As noted by March and Simon, if relatedness occurs on several dimensions, then firms may need to trade off improvements along one dimension with reductions along others.

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[7] This is the case if $b > 7 + 4\sqrt{3}$.

5. Learning about interactions between components

This section extends the model to analyze 'modular' design processes. Modular design is characterized by design groups separated by standardized interfaces that govern how the subsystems are to fit together into a whole product. This feature allows the firm to 'mix-and-match' components from various versions of the product. Baldwin and Clark emphasize that modular design is different from a simple division of design labor. They stress that merely dividing labor does not require the standardized interfaces and flexible architecture of modular design systems.

How might a firm gain from developing standardized interfaces to 'mix-and-match' components and subsystems? In the model developed above, a firm could receive a negative research shock to a given product design group (perhaps because a crucial new technology failed to meet expectations). In this case, the firm may prefer to use the previous version of that subsystem. However, unless the new and old versions of the subsystem are designed to meet the same interface specification, the old version may not fit the new versions of the other subsystems. Thus, modular design allows firms to use only those new subsystems that represent improvements over the old. Baldwin and Clark show that the ability to combine old and new increases the firm's R&D productivity. Modularity can also increase product variety. If combining old and new versions of various subsystems results in distinct versions of the product, then modular design reduces the cost of enhancing the variety of a product line.

Baldwin and Clark's model balances these two advantages of modular design against the cost of testing the many potential products generated by modular design processes. I explore another potential advantage of modular design by showing that this process of 'mixing-and-matching' can aid the firm in learning about the interactions between components. In particular, I assume that the firm is initially uncertain as to the values of the complementarity parameters. I compare the firm's ability to infer the δ_{ij}'s from the performance of successive generations of the product under three types of design processes:

1. Monolithic design with no division of design labor;
2. Monolithic design with a division of design labor;
3. Modular design.

I show that, using monolithic design with no division of labor, the firm is unable to gain information regarding the interactions between components. Using monolithic design with a division of labor allows the firm to gain some information as to the relative magnitudes of the complementarity parameters. However, the firm can gain even finer information using a modular design strategy. These results formalize a claim of Baldwin and Clark, who write "Generally, a design team taking a 'monolithic' approach will not attempt to break down the product to see what principles govern its performance. As a result, such teams achieve only a relatively superficial understanding of the product and its production system."

I assume the firm can observe the product's aggregate performance, X, but that it does not observe the individual x_i's. That is, measuring aggregate product performance (perhaps through testing) is feasible, but attributing product performance to the quality of any one component is difficult. Also, while the firm does know the mean and variance of the research shocks ϵ_i conditional on the size of the design groups, the firm cannot observe individual values of ϵ_i.

I first analyze knowledge obtained via a monolithic design strategy without division of labor. The firm observes the performance of the initial version of the product

$$X_0 = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \delta_{ij} x_i x_j.$$

The monolithic design process then subjects each x_i to a random additive shock ϵ_i. The firm then observes

$$X_1 = X_0 + \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \delta_{ij}(x_i \epsilon_j + x_j \epsilon_i + \epsilon_i \epsilon_j). \tag{8}$$

Note that in observing $X_1 - X_0$, the firm is unable to learn anything about the relative magnitudes of the complementarity parameters. Since the firm does not know the x_i's or the ϵ_i's, it cannot disentangle the effects of the δ_{ij}'s. If $X_1 - X_0$ is larger than expected, the firm knows only that the large x_i's and ϵ_i's were paired with the large δ_{ij}'s.

Alternatively, consider a firm that develops the second version of the product using a monolithic strategy with a division of design labor. I assume the firm first observes X_0 and then selects an arbitrary design partition: $\mathcal{G}_1 = \{1, \ldots, N/2\}$, $\mathcal{G}_2 = \{N/2+1, \ldots, N\}$.[8]

The firm observes the improvement in the product's performance $X_1 - X_0$, which I rewrite from Eq. (8) as:

$$\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \delta_{ij}(x_i \epsilon_j + x_j \epsilon_i) + \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \delta_{ij}(\epsilon_i \epsilon_j).$$

Note that the first term in this expression is not affected at all by the firm's chosen design partition. The expectation of the second term does, however, depend on this decision.

$$E\left[\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \delta_{ij}(\epsilon_i \epsilon_j)\right] = \rho_{\mathrm{w}} \left(\sum_{i=1}^{N/2-1} \sum_{j=i+1}^{N/2} \delta_{ij} \sigma_i \sigma_j + \sum_{i=N/2+1}^{N-1} \sum_{j=i+1}^{N} \delta_{ij} \sigma_i \sigma_j \right) + \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \delta_{ij} \left\{ \mu\left(\frac{N}{2}\right) + \rho_{\mathrm{a}} \sigma_i \sigma_j \right\}.$$

If the performance of the product is high relative to expectations, then it is likely that

$$\sum_{i=1}^{N/2-1} \sum_{j=i+1}^{N/2} \delta_{ij} \sigma_i \sigma_j + \sum_{i=N/2+1}^{N-1} \sum_{j=i+1}^{N} \delta_{ij} \sigma_i \sigma_j$$

is relatively large. This would imply that the selected partition is capturing many of the

[8] Without loss of generality, I assume the product consists of an even number of components.

important interactions between components. Hence, a firm trying to update beliefs regarding the complementarity parameters is able to gain information from observing X_1 if it uses an arbitrary division of design labor. The intuition for this result is that, by partitioning the components and designing a new version of the product, the firm learns whether the chosen division of labor captures or ignores important interactions between components.

A firm using a modular design process is able to gain even finer information regarding the complementarity parameters. Suppose the firm develops interfaces that allow it to mix and match modules from different generations of the product. The firm can match the new version of some modules with old versions of others. This matching captures a different set of interactions between the components and allows the firm to draw better inferences.

I denote the resulting product performance when the new (old) version of $\mathcal{G}_1$ is matched with the old (new) version of $\mathcal{G}_2$ by X_a (X_b). In comparing the performances of the various versions of the product, the firm can construct

$$X_1 - X_a - X_b + X_0 = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \delta_{ij}\epsilon_i\epsilon_j I[G(i) \neq G(j)]$$

which depends only on the interactions between components that are not currently grouped in the same module. The larger this quantity, the more likely the current grouping is not capturing all the important linkages between components. In addition to obtaining some information regarding the interactions between components grouped together (from $X_1 - X_0$), the firm receives information from $X_1 - X_a - X_b + X_0$ on the interactions between components that are not grouped together. The construction of X_a and X_b provides incremental information to the firm because X_a contains information regarding the interactions between components in $\mathcal{G}_1$, but not about interactions between components in $\mathcal{G}_2$. By subtracting X_a and X_b from X_1, the firm can focus attention on interactions captured by neither module.

In this analysis, I have assumed that the firm has no information whatsoever regarding the interactions between components. This assumption may not apply well to the notions of physical fit – an auto firm is likely to be well aware that piston rings and ashtrays have no necessary physical connection. However, in contemplating what Clark and Fujimoto call ‘external product integrity,’ a firm is likely to consider the relationship of the ashtray to the design of other dashboard components and the relationship of the dashboard to engine characteristics. While extent of physical fit may be easy to ascertain, the extent to which components interact to affect the way a product is perceived by a customer may be much more obscure.

Given this discussion, it seems apparent that modular design processes may confer important advantages upon a firm that implements them. While I do not model them explicitly, two types of costs may inhibit the firm from choosing processes of this type. First, it may be costly to determine X_a and X_b separately from X_0 and X_1. This cost of testing is considered explicitly by Baldwin and Clark. In the model presented here, it is clear that high testing costs lead firms to choose monolithic design strategies more frequently. Second, the firm may find it costly to develop interfaces that allow ‘mixing-and-matching' of modules. If new interfaces must be developed each time a firm changes its modularization strategy, then a firm may choose to retain a sub-optimal modularization strategy if the costs of developing interfaces outweigh the benefits from improved modularization. A fixed cost of interface development could induce path dependence in modularization.

6. Computational feasibility of optimal design partitions

Given the substantial informational requirements of modular design and the role of modular design itself in uncovering the information needed to develop an effective modular design partition, it would seem unlikely that a firm could ever hope to uncover an optimal modular design partition for a complex product. In this section, I demonstrate that, even if the complementarity parameters are known, the problem of partitioning a large set of components into design modules is likely to be difficult.[9]

To demonstrate this formally, I apply the theory of NP-completeness from computer science.[10] The set of NP-complete problems includes such well-known examples of 'difficult' combinatoric problems as the traveling salesman problem and the knapsack problem. No known algorithm solves these problems in a time period that is a polynomial function of the size of the problem. For these problems, complexity is thought to increase exponentially with size. While computer scientists have been unable to prove that solving time is exponentially related to size, large instances of these problems are generally thought to be intractable.

To demonstrate a particular problem is NP-complete, I must show that it is isomorphic to a problem that is known to be NP-complete. I begin by selecting a special case of the firm's optimization problem listed in Eq. (2). I demonstrate that the special case is NP-complete, which then implies that the more general problem cannot be solved in polynomial time by a known algorithm.

The firm designs a product with N components. Let $\sigma_i^2 = \sigma^2$ for all $i \in \{1, \ldots, N\}$ and suppose the function mapping the number of components in each module to expected research productivity is characterized by

$$\mu(M) = \begin{cases} 1 & \text{if } M \leq K \\ -\infty & \text{otherwise,} \end{cases}$$

where $K \geq 3$. This states that it is prohibitively costly to design modules with more than K components. The firm's problem is then to partition the components of the product into modules of at most K components in order to minimize coordination loss.

Substituting for μ, I rewrite Eq. (2) to obtain:

$$\max_{\mathcal{G}_1,\ldots,\mathcal{G}_m} X_0 + \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \delta_{ij} \{ x_i + x_j + 1 + \rho_{\mathrm{w}} \sigma^2 I[G(i) = G(j)] \}.$$

[9] In focusing on the computational aspects of large problems with complementarities, the discussion in this section is related to recent work by Page (1997). Page derives an efficient algorithm for finding the optimal collection of public projects when the projects are complementary.

[10] Garey and Johnson (1978) is the standard reference.

Dropping terms that do not depend on the firm's decision as to which components to place in which design groups leaves

$$\max_{\mathcal{G}_1,\ldots,\mathcal{G}_m} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \delta_{ij} I[G(i) = G(j)] \tag{9}$$

The firm's objective in partitioning components is to maximize the sum of the complementarity parameters between the components placed in the same module, or, equivalently, to minimize the sum of the across-module complementarity parameters.

Proposition 3. *Problem (9) is NP-complete. (See Appendix for proof).*

The proposition implies that, even if the complementarity parameters are known, it is likely to be extremely difficult to devise an optimal design partition. A firm with a complex product has no feasible way of verifying that a given design partition is optimal. Depending on the costs of developing new interfaces, a firm may wish to make continuous adjustments to its design partition as better-performing alternatives become known.

As in the previous section, it is important to recall Clark and Fujimoto's two senses in which components must fit. If physical fit were the only important consideration, then only interactions between physically adjacent components would matter. This would place a great deal of additional structure on the firm's problem and make it easier to solve. However, when components from non-adjacent parts of the product interact to affect customers' perceptions, then the firm must solve the most general version of Eq. (9).

7. Conclusions

This paper has combined two strands of recent literature to study a firm's decision as to how to partition the work of product design when the components of the firm's product are complementary. I apply results on supermodular functions to add to the literature on modular design by explicitly considering how the interactions between components might affect the process of partitioning components into research groups. I show that optimal design partitions depend on the costs of across- and within-design group communication and the amount of uncertainty the firm faces in designing the next version of a given component. I confirm Baldwin and Clark's conjecture that a firm is likely to gain more information regarding interactions between components using a modular design strategy as opposed to a monolithic design strategy. Finally, I use the theory of NP-completeness to show that solving general instances of the product design partition problem is likely to be difficult.

Future work may be able to extend this model usefully in several areas. First, the model could be applied to study how design partitions in firms evolve over time. In particular, it would be interesting to compare the costs of developing new standardized interfaces with the benefits of achieving better product design partitions. Are real product design partitions strongly path dependent? Second, the current model could be joined with other theories of organization to study interactions between the technological considerations analyzed here and factors arising from a firm's human resource management policies. Might the firm's choice of design partitions affect the incentives provided to design workers? How many and what type of employees are needed to staff different sized product development teams? Third, at a more technical level, the model could be extended to consider the effects of interactions between larger groups of components. What insights could be gained from relaxing the assumption that complementarity parameters apply pairwise? Finally, the model could be extended to study market relationships between firms competing to provide different modules of an 'open' system. How are components allocated to modules across firm boundaries? An interesting comparison might be that between design partitions reached through market means and those selected by a single decision maker.

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Appendix

Proof of Proposition 1. Normalize the product's performance at time 0 to $X_0 = 0$ with $x_{10} = \cdots = x_{N0} = 0$. I first demonstrate that m is a non-increasing function of γ_a. Sufficient conditions for the comparative static result are that the firm's objective function is supermodular in (m, $-\rho_w$, ρ_a) and has increasing differences in (m, $-\rho_w$, ρ_a; $-\gamma_a$) (see Milgrom and Shannon (1994) Theorem 5). Note that Eq. (7) implies the firm's objective function has increasing differences in (m, $-\rho_w$, ρ_a; $-\gamma_a$). To demonstrate supermodularity in (m, $-\rho_w$, ρ_a), it is sufficient to show

$$F(m_1, \rho_w, \rho_{a1}) - F(m_2, \rho_w, \rho_{a1}) > F(m_1, \rho_w, \rho_{a2}) - F(m_2, \rho_w, \rho_{a2}) \quad (A.1)$$

for all $m_1 > m_2$ and $\rho_{a1} > \rho_{a2}$ and

$$F(m_1, \rho_{w1}, \rho_a) - F(m_2, \rho_{w1}, \rho_a) > F(m_1, \rho_{w2}, \rho_a) - F(m_2, \rho_{w2}, \rho_a) \quad (A.2)$$

for all $m_1 > m_2$ and $\rho_{w1} < \rho_{w2}$, where F denotes the firm's objective function.[11]

To demonstrate Eqs. (A.1) and (A.2), I select arbitrary $m_1 > m_2$. Denote by $\mathscr{G}_1, \ldots, \mathscr{G}_{m_1}$ the optimal design partition when m_1 groups have been chosen and let G be the function mapping components to design groups for this partition. Let $\mathscr{H}_1, \ldots, \mathscr{H}_{m_2}$ be the optimal design partition when m_2 groups have been chosen and let H be the associated function

[11] Note that $\partial^2 F/\partial\rho_a\partial\rho_w = 0$.

mapping components to design groups. Without loss of generality, I can restrict attention to m_2 for which

$$\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\delta_{ij}\sigma_i\sigma_j I[G(i)=G(j)] > \sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\delta_{ij}\sigma_i\sigma_j I[H(i)=H(j)] \tag{A.3}$$

for all $m_1 > m_2$. If there exists an $m_1 > m_2$ for which this statement is not true, then a design partition with m_2 groups is never optimal, since the partition with m_1 groups has both higher research productivity and smaller coordination loss. I write the left hand side of Eq. (A.1) as

$$\left(\mu\left(\frac{N}{m_1}\right)^2 - \mu\left(\frac{N}{m_2}\right)^2\right)\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\delta_{ij} - (\rho_w - \rho_{a1})$$
$$\times\left(\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\delta_{ij}\sigma_i\sigma_j I[G(i)=G(j)] - \sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\delta_{ij}\sigma_i\sigma_j I[H(i)=H(j)]\right).$$

The right hand side of Eq. (A.1) is identical except ρ_{a2} replaces ρ_{a1}. Simplify by subtracting $F(m_1, \rho_w, \rho_{a2}) - F(m_2, \rho_w, \rho_{a2})$ from both sides:

$$(\rho_{a1} - \rho_{a2})\left(\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\delta_{ij}\sigma_i\sigma_j I[G(i)=G(j)] - \sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\delta_{ij}\sigma_i\sigma_j I[H(i)=H(j)]\right) > 0.$$

Since by assumption $\rho_{a1} > \rho_{a2}$, this statement holds if and only if Eq. (A.3) is true. This establishes Eq. (A.1).

Similar simplifications yield that Eq. (A.2) is equivalent to

$$(\rho_{w2} - \rho_{w1})\left(\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\delta_{ij}\sigma_i\sigma_j I[G(i)=G(j)] - \sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\delta_{ij}\sigma_i\sigma_j I[H(i)=H(j)]\right) > 0.$$

Since by assumption $\rho_{w2} > \rho_{w1}$, Eq. (A.3) implies Eq. (A.2)

Since the firm's objective function is supermodular in $(m, -\rho_w, \rho_a)$ and has increasing differences in $(m, -\rho_w, \rho_a; -\gamma_a)$, I conclude that m is non-increasing in γ_a. Note also that Eq. (6) implies that the firm's objective function has increasing differences in $(m, -\rho_w, \rho_a; \gamma_w)$. Together with the supermodularity result demonstrated above, this means that m is non-decreasing in γ_w. □

Proof of Proposition 2. Let $\mathscr{G}$ be a product design partition with the property that $G(i) \neq G(j)$. I show that, for any $\mathscr{G}$, there exists a partition $\mathscr{H}$ with the property that $H(i) = H(j)$ and a B' such that if $b > B'$, then $\mathscr{H}$ weakly dominates $\mathscr{G}$.

Without loss of generality, let $i = 1$ and $j = 2$. I consider two cases. First, suppose $\mathscr{G}$ has the property that $G(1) = \{1\}$ and $G(2) = \{2\}$. Let $\mathscr{H}$ be a partition that is identical to $\mathscr{G}$ except that $G(1)$ and $G(2)$ are replaced by $\{1, 2\}$. I subtract the firm's payoff when selecting partition $\mathscr{G}$ from the firm's payoff when selecting $\mathscr{H}$ and simplify to obtain

$$\delta_{12}\{(x_1 + x_2)(\mu[2] - \mu[1]) + (\mu[2]^2 - \mu[1]^2) + b\sigma^2(\rho_w - \rho_a)\}.$$

The only term in the expression containing b is

$$\delta_{12} b\sigma^2(\rho_\mathrm{w} - \rho_\mathrm{a})$$

which is positive. The other terms in the expression are finite, since $\mu(2) > -\infty$. Hence, there exists B' such that if $b > B'$, the firm prefers $\mathscr{H}$ to $\mathscr{G}$.

Now consider a second case. Let $\mathscr{G}$ have the property that either $G(1)$ or $G(2)$ is not a singleton. Without loss of generality, suppose that $G(1)$ is not a singleton and suppose $\{3\} \subset G(1)$. Let $\mathscr{H}$ be a partition that is identical to $\mathscr{G}$ except that $G(1)$ is replaced by $G(1) \cup \{2\} \backslash \{3\}$ and $G(2)$ is replaced by $G(2) \cup \{3\} \backslash \{2\}$. If the firm's payoff under $\mathscr{G}$ is $-\infty$, then the firm's payoff under $\mathscr{H}$ is $-\infty$ as well, so the firm is indifferent between the two. Suppose the firm's payoff under $\mathscr{G}$ is finite. Subtract the firm's payoff when selecting partition $\mathscr{G}$ from the firm's payoff when selecting $\mathscr{H}$ and simplify to obtain

$$\begin{aligned}
&\sum_{i \epsilon G(1)\backslash\{3\}} [\delta_{i3}\{(x_i + \mu[\#G(1)])(\mu[\#G(2)] - \mu[\#G(1)]) + (\rho_\mathrm{a} - \rho_\mathrm{w})\sigma_i\sigma_3\} \\
&\quad + \delta_{i2}\{(x_i + \mu[\#G(1)])(\mu[\#G(1)] - \mu[\#G(2)]) + (\rho_\mathrm{w} - \rho_\mathrm{a})\sigma_i\sigma_2\}] \\
&\quad + \sum_{j \epsilon G(2)\backslash\{2\}} \left[\delta_{j2}\left\{\left(x_j + \mu[\#G(2)]\right)(\mu[\#G(1)] - \mu[\#G(2)]) + (\rho_\mathrm{a} - \rho_\mathrm{w})\sigma_j\sigma_2\right\}\right. \\
&\quad \left. + \delta_{j3}\left\{\left(x_j + \mu[\#G(2)]\right)(\mu[\#G(2)] - \mu[\#G(1)]) + (\rho_\mathrm{w} - \rho_\mathrm{a})\sigma_j\sigma_3\right\}\right].
\end{aligned}$$

There are several terms in this expression that contain $\sqrt{b}$, but only

$$\delta_{12} b\sigma^2(\rho_\mathrm{w} - \rho_\mathrm{a})$$

contains b. Note that this term is positive. The other terms in the expression are all finite, so the limit of the expression at b grows large is $+\infty$. I therefore have that there exists B' such that if $b > B'$, the firm prefers $\mathscr{H}$ to $\mathscr{G}$.

Since the number of possible design group partitions is finite there must exist a B such that if $b > B$, the optimal design group partition has the property that $G(1) = G(2)$. □

Proof of Proposition 3. I proceed by analyzing the dual of Eq. (9):

$$\min_{\mathscr{G}_1,\ldots,\mathscr{G}_m} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \delta_{ij} I[G(i) \neq G(j)]. \tag{A.4}$$

I compare this problem to the following NP-complete problem:

Graph Partitioning (see Garey and Johnson, 209 pp.).
Instance: Graph $G = (V, E)$, weights $w(v) \in Z^+$ for each $v \in V$, lengths $l(e) \in Z^+$ for each $e \in E$, and positive integers K and J.
Question: Is there a partition of V into disjoint sets $V_1, \ldots, V_m$ such that $\sum_{v \in V_i} w(v) \leq K$ for $1 \leq i \leq m$ and such that if $E' \subseteq E$ is the set of edges that have their two endpoints in two different sets V_i, then $\sum_{e \in E'} l(e) \leq J$?

Garey and Johnson report that this problem is still NP-complete for fixed $K \geq 3$ even if all w and l are one.

Consider the instances of Graph Partitioning for which $w(v_i) = 1$ for all i. I argue that Eq. (A.4) is isomorphic to this class of problems. To show this, I demonstrate that for any instance of Eq. (A.4), there is a corresponding instance of Graph Partitioning. I then show that for any instance of Graph Partitioning, there is a corresponding instance of Eq. (A.4).

Consider an arbitrary instance of Eq. (A.4). For each component i, construct a vertex v_i. Let $w(v_i) = 1$ for all i. If $\delta_{ij} > 0$, then let G contain an edge connecting v_i and v_j. Denote this edge by e_{ij} and let $l(e_{ij}) = \delta_{ij}$. Then, if the partition of components $\mathcal{G}_1,\ldots,\mathcal{G}_m$ is a solution to Eq. (A.4), then the partition of the corresponding vertices $V_1,\ldots,V_m$ is the partition of G that minimizes $\sum_{e\in E'} l(e)$. Hence, a solution to Eq. (A.4) implies a solution to the corresponding instance of Graph Partitioning.

Now consider an arbitrary instance of Graph Partitioning with $w(v_i) = 1$ for all i. For each vertex v_i, construct a component i. If G does not contain an edge between vertices v_i and v_j, then set $\delta_{ij} = 0$. If G does contain an edge between v_i and v_j, then set $\delta_{ij} = e_{ij}$. Then, if the partition of vertices $V_1,\ldots,V_m$ is a solution to Graph Partitioning, then the partition of the corresponding components $\mathcal{G}_1,\ldots,\mathcal{G}_m$ solves Eq. (A.4). □

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